Development and use of a vane device for boundary-layer measurements

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A hinged vane and a sensitive electrical system for recording the motion of the vane have been developed for the observation of fluctuating y-components of velocity in boundary layers. An approximate theory of the natural oscillations of such vanes is presented and experimentally verified. Using vanes as resonant detectors, measurements have been made of oscillations injected into the laminar boundary layer on a flat plate in a wind tunnel with 0·3 % free-stream turbulence. Points on the neutral Tollmien–Schlichting curve have thereby been obtained which lie close to the theoretical neutral curve.

1. Introduction

The importance of the introduction by Prandtl in 1904 of the concept of the boundary layer is shown, not only by the large part which the concept has played in the development of fluid dynamics, but also by the great variety of phenomena which the layer itself presents for investigation. A new approach to the study of these phenomena derives from the mathematical analysis of the periodic properties of the laminar boundary layer by Tollmien (1929, 1935), Schlichting (1933, 1935), Lin (1945–6) and Shen (1954), and from the experimental verification of these properties by Schubauer & Skramstad (1947) and Schubauer & Klebanoff 1955). Particular interest attaches to the demonstration by Schubauer *et al.* that under low free-stream turbulence conditions there is a close connexion between the occurrence of these natural oscillations and the onset of boundary-layer turbulence. This connexion has not yet been established theoretically, and the difficulties of doing so seem great. We are therefore forced to concentrate attention on the experimental investigation of the transition to turbulence.

In view of the large part which appears to be played by periodic fluctuations in exciting the transition to turbulence, it seemed to us that a detailed study of the periodicities occurring in the initial stages of turbulent flow might be useful, and that for this purpose it might be possible to develop a vane-type instrument which would respond to the transverse component of velocity in the boundary layer, and thereby record a feature of the disturbances different from that recorded by the simple hot-wire anemometer. The present paper reports the progress which has been made with this project.

2. Description of the wind tunnel

An 18 in. octagonal open-circuit wind tunnel, a modification of N.P.L. Design no. A 155, was constructed for the work. An extra section 3 ft. 6 in. long was introduced before the contraction to accommodate smoothing screens, and a further section 4 ft. long was added between the contraction and the working section to allow the turbulence to become isotropic after the contraction. The contraction ratio was 3·16. A four-bladed fan was used, producing an air-speed of 100 ft./sec in the working section with a fan-speed of 2800 r.p.m., a Ward–Leonard system being used for speed control. A vertical flat plate, 6 ft. × 18 in., of $\frac{1}{4}$ in. Perspex, was mounted centrally in the tunnel; its leading edge, worked to a symmetrical knife edge, was located 1 ft. upstream of the beginning of the working section.

Particular care was taken in assembling the tunnel to minimize vibration. The diffuser was coupled with canvas to the working section, and all tunnel and motor units were mounted on concrete blocks insulated from the concrete floor with layers of felt.

The free-stream turbulence was measured by the thermal diffusion method due to Taylor (1935). The optimum number of smoothing screens was found to be two, and the minimum transverse component of turbulence was 0·3 %. The noise level in the tunnel room rose to 107 decibels at a fan speed of 2800 r.p.m., the resulting acoustic velocities being equivalent to 0·03 % turbulence. False walls of $\frac{1}{16}$ in. Perspex were introduced, beginning in the contraction, to obtain zero pressure change in the working section.

3. Preliminary work on vanes

While the tunnel was under construction, the preliminary work on vanes was carried out. For this, a miniature wind tunnel was constructed with a working cross-section of 3 in. × 3 in., a contraction ratio of 16, and a maximum wind speed of about 40 ft./sec.

From a scrutiny of the recordings made by Schubauer & Klebanoff (1955), it appeared that, using wind-speeds of 80 ft./sec, disturbances with frequencies up to 1000 c/s and periodic lengths down to 1 in. might be of interest. To record such disturbances it would be necessary to employ vanes with a chord not greater than $\frac{1}{2}$ in.

Rectangular vanes were made from silvered microscope coverslip, of 0·007 in. thickness; their aspect ratio varied between $\frac{2}{3}$ and 1, and their chord from a quarter to fully half an inch. U-shaped hinges made from wire of 0·001 in. diameter were glued on the vanes near the extremities of the leading edge. When a vane was under test, it was supported by the hinges on a vertical suspension wire of 0·007 in. diameter. The suspension wire passed through small holes in the top and bottom of the tunnel; it was kept under tension by attaching weights to its lower end; and it was clamped above and below the tunnel in such a manner as to insulate it from tunnel vibration. A small bead of glue was put on the suspension wire to prevent the vane from slipping downwards. When the vane did not move sufficiently freely, a small air jet was used to make it rotate continuously for a few minutes.

The movements of the vane under the influence of the air stream were observed by the optical lever method—a beam of light was reflected from the silvered surface of the vane into the aperture of a moving film camera. The vanes were thus found to possess natural frequencies of oscillation in the frequency range 30–1000 c/s. A theory of the natural frequencies of oscillation of vanes freely suspended on wires under tension was therefore developed and checked experimentally.

4. Approximate theory of the oscillations of a rectangular vane freely hinged at the leading edge

Figure 1 shows a sketch of the vane and suspension wire, and indicates the rectangular co-ordinate system used in the analysis. The origin O is at one fixed end of the wire. The x- and z-axes are taken parallel respectively to the direction of the air flow U and the direction of the undisturbed wire. The mass, chord, and span of the vane are represented respectively by m, $2a$, and s, the line density, the tension, and the free length of the wire by ρ', T and l. The centre of gravity of the vane lies in the plane $z = \frac{1}{2}l$. The density of the air stream is denoted by ρ. The displacement of the wire in the y-direction is represented by η.

An analysis of the oscillatory motion of the vane is required under the following assumptions:

(*a*) That the air stream has an effectively constant x-component of velocity U and a variable y-component v such that v/U is small.

(*b*) That the variable angle α between the vane and the x-axis is comparable with v/U in magnitude.

(*c*) That for displacements of the wire parallel to the y-axis, the reaction of the vane on the wire is more important than the aerodynamic forces on the wire.

(*d*) That the motion of the system parallel to the x-axis is negligible.

The vane moves under the action of L and D, the forces of lift and drag, and of F_x and F_y, the x- and y-components of the reaction at the hinge. The moment of the aerodynamic forces about an axis parallel to the z-axis through the centre of gravity of the vane is denoted by M. Taking $C_{L\alpha}$ and M_α as the coefficients of L and M per radian angle of incidence, and taking $v = 0$, we have

$$\left.\begin{aligned} L &= C_{L\alpha}\rho a s U^2\alpha \;= CU^2\alpha, \\ M &= 2M_\alpha \rho a^2 s U^2\alpha = C'aU^2\alpha. \end{aligned}\right\} \tag{1}$$

When v is not zero, the angle of incidence of the stream on the vane becomes $\alpha+\tan^{-1}(v/U)$. Then $L = C(U^2+v^2)(\alpha+\tan^{-1}(v/U))$ and the y-component of L is $L_y = LU/(U^2+v^2)^{\frac{1}{2}}$. In accordance with assumptions (*a*) and (*b*) above, we neglect second and higher powers of v/U so that

$$L_y = CU^2(\alpha+v/U).$$

Similarly,

$$M = C'aU^2(\alpha+v/U). \tag{2}$$

The drag coefficient C_D may be expressed as the sum of the drag coefficient at zero incidence C_{D0} and the induced drag $2aC_{L\alpha}^2\alpha^2/\pi s$ (cf. Campbell, Blanks & Leaver 1956). The ratio of the y-components of drag and lift acting on a vane in the disturbed flow is then

$$\frac{D_y}{L_y} = \left[\frac{C_{D0}}{C_{L\alpha}(\alpha + v/U)} + \frac{2a}{\pi s} C_{L\alpha}\left(\alpha + \frac{v}{U}\right)\right]\frac{v}{U}$$
$$\simeq \frac{C_{D0}}{2C_{L\alpha}} + \frac{2C_{L\alpha}\alpha^2}{\pi}.$$

Using the Blasius value for C_{D0}, the first term is of the order of 10^{-2}, and taking $\alpha \sim 0{\cdot}01$, the second term is $\sim 10^{-4}$. Thus the drag force does not contribute appreciably to the lateral motion of the vane.

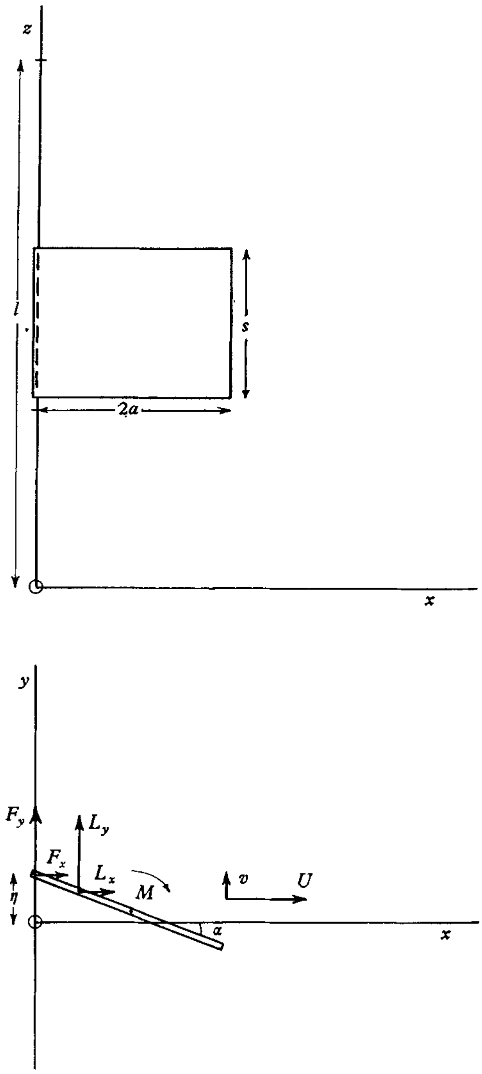

FIGURE 1. Sketch of the vane and suspension wire indicating the system of co-ordinates.

The linear motion of the centre of gravity of the vane is then governed by

$$CU^2(\alpha + v/U) + F_y + r_y = m(\ddot{\eta}_{z=\frac{1}{2}l} - a\ddot{\alpha}), \tag{3}$$

and the angular motion about an axis parallel to the z-axis through the centre of gravity of the vane by

$$C'aU^2(\alpha + v/U) + aF_y + ar_M = \tfrac{1}{3}ma^2\ddot{\alpha}, \tag{4}$$

where ar_M and r_y are respectively the resistive couple and the y-component of resistive force on the vane.*

According to this analysis, the effect of the transverse velocity component v is to exert on the vane a force CUv, and a moment $C'aUv$. If we suppose v to be a periodic disturbance with an arbitrary frequency, then equations (3) and (4) are the differential equations of motion for forced oscillations of an oscillatory system driven at that frequency. To determine the resonance frequencies of the oscillatory system we write $v = 0$ and omit the resistive terms in these equations. The resonance frequencies are therefore given by

$$CU^2\alpha + F_y = m(\ddot{\eta}_{z=\frac{1}{2}l} - a\ddot{\alpha}), \tag{5}$$

$$C'aU^2\alpha + aF_y = \tfrac{1}{3}ma^2\ddot{\alpha}. \tag{6}$$

Considering now the motion of the wire, and taking account of assumptions (c) and (d) above,

$$F_y = T\left(\frac{\partial \eta_2}{\partial z} - \frac{\partial \eta_1}{\partial z}\right)_{z=\frac{1}{2}l}, \tag{7}$$

where η_1 and η_2 represent the displacements of the wire for $z \leqslant \frac{1}{2}l$ and $z \geqslant \frac{1}{2}l$ respectively. Also, writing $c^2 = T/\rho'$, the motion of the wire obeys the equation

$$c^2\frac{\partial^2 \eta}{\partial z^2} = \frac{\partial^2 \eta}{\partial t^2}, \tag{8}$$

subject to the boundary conditions $\eta = 0$ at $z = 0$ and $z = l$, and $\eta_1 = \eta_2$ at $z = \frac{1}{2}l$.

Equation (8) and its boundary conditions are satisfied by a solution of the form

$$\left.\begin{aligned} \eta_1 &= A \sin kz \sin \omega t, \\ \eta_2 &= A \sin k(l-z) \sin \omega t \quad (kc = \omega), \end{aligned}\right\} \tag{9}$$

making

$$F_y = -2AkT \cos \tfrac{1}{2}kl \sin \omega t. \tag{10}$$

On substituting in equations (5) and (6) for $\ddot{\eta}$ and F_y from (9) and (10), and eliminating α and $\ddot{\alpha}$, the equation for ω is obtained in the form

$$\frac{\omega}{2\rho' c}\tan\frac{\omega l}{2c} = \frac{4ma\omega^2 - 3(C - C')\,U^2}{m(ma\omega^2 + 3C'U^2)}. \tag{11}$$

In this equation, the parameters of the vane appear only on the right-hand side and those of the wire only on the left-hand side. This suggests that, by suitable choice of the values of these parameters, the frequencies of the coupled system may be made characteristic either of the vane or of the wire.

* The dissipative terms are required in the complete equations of motion of an oscillatory system. They are not required in the subsequent analysis, but they determine the 'breadth' of the response curves shown in figure 3.

Equation (11) may be solved graphically if the values of C and C' are known, but some uncertainty attaches to these constants on account of the interaction between the air stream and the wire. It is sufficient for present purposes to examine the general nature of the solution and to do this we write equation (11) in another form, making the following substitutions:

$$\omega l/2c = \theta, \quad m/4\rho' l = G \quad \text{(a coupling constant)},$$
$$A^2 = 3(C-C')\,U^2 l^2/16mac^2,$$
$$B^2 = 3C'\,U^2 l^2/4mac^2.$$

It is necessary to be specific about the sign of A^2, so we write $C' = 0{\cdot}6C$. This is equivalent to making the assumption that the lift force acts at a distance from

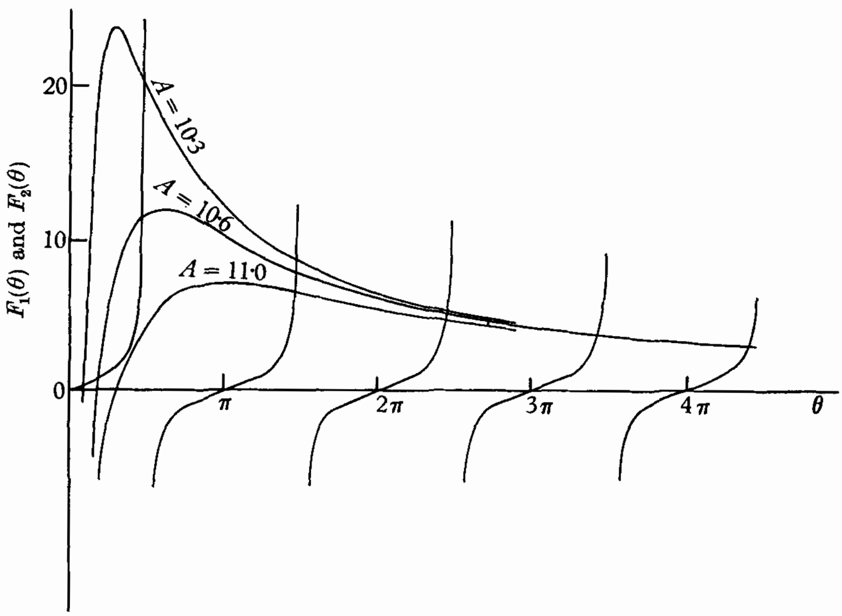

FIGURE 2. Graphical solution of equation (12).
$F_1(\theta) = \tan\theta$; $F_2(\theta) = (\theta^2 - A^2)/[G\theta(\theta^2 + 6A^2)]$; $G = \frac{1}{40}$.

the leading edge of $\frac{1}{5}$ of the chord. This assumption is in good agreement with experimental work on rectangular wings of small aspect ratio, with a profile similar to the vane (for example, the observations of Campbell *et al.* (1956) on their profile F). It follows that A^2 is positive and also that $B^2 = 6A^2$. Equation (11) then becomes

$$\tan\theta = \frac{(1/G)(\theta^2 - A^2)}{\theta(\theta^2 + 6A^2)}. \tag{12}$$

The graphs of

$$F_1(\theta) = \tan\theta \quad \text{and} \quad F_2(\theta) = \frac{(1/G)(\theta^2 - A^2)}{\theta(\theta^2 + 6A^2)}$$

are shown in figure 2. $F_2(\theta)$ has been calculated for three values of A, namely 0·3, 0·6 and 1·0 radians, and for one value of G, namely 1/40. As the ordinates of $F_2(\theta)$ are inversely proportional to G, the effect of changing this constant is obvious. The first branch of $F_1(\theta)$ makes at most two intersections with $F_2(\theta)$. The corresponding solutions for θ will be referred to as θ_0 and θ_1 with $\theta_0 \leqslant \theta_1$. The second and subsequent branches of $F_1(\theta)$ each make one intersection with $F_2(\theta)$, and the corresponding solutions for θ will be referred to as θ_2, θ_3, etc.

For an unloaded wire the values of θ are given by $(n-\frac{1}{2})\pi$; $n = 1, 2, 3, \ldots$. It is evident that as the coupling constant G diminishes, and the ordinates of $F_2(\theta)$ rise, θ_n tends to $(n-\frac{1}{2})\pi$. The frequencies $f_n = \omega_n/2\pi = c\theta_n/\pi l$ will therefore be referred to as wire frequencies. In the low coupling limit,

$$f_n^2 l^2 \rho'/T = (n-\tfrac{1}{2})^2. \tag{13}$$

The intersection giving θ_0 exists only for low values of A and G. To a first approximation when A is small, $\theta_0 \simeq A + 3{\cdot}5GA^3$, and in the low coupling limit $\theta_0 = A$ or

$$4ma\omega_0^2 = 3(C-C')\,U^2. \tag{14}$$

This equation is in fact the solution of equations (5) and (6) when $\ddot{\eta} = 0$. The equation leads to a unique value of the frequency, and since this frequency depends only on vane parameters, we interpret $f_0 = \omega_0/2\pi = c\theta_0/\pi l$ as a vane frequency. If in equation (14) we substitute $C' = 0{\cdot}6C = 0{\cdot}6C_{L\alpha}\rho as$, then, in the low coupling limit

$$mf_0^2/\rho s U^2 = 0{\cdot}3C_{L\alpha}/4\pi^2. \tag{15}$$

5. Experimental tests of the theory of natural oscillations of vanes

Using the procedure described in § 3, with $U = U_0$, measurements were made of the natural frequencies of oscillation of a number of vanes under different suspension conditions. The photographic records usually showed two natural frequencies; the main feature of the trace was a low-frequency oscillation of 2 or 3 mm amplitude somewhat distorted by the turbulence in the tunnel, but, superimposed on this, a higher frequency ripple of much smaller amplitude could usually be seen. The low frequency was insensitive and the high frequency sensitive to change of tension on the suspension. The lower frequency was therefore interpreted as a vane frequency and the higher as a wire frequency. Typical results for two vanes (A and B) are shown in table 1.

As a check on this interpretation of the frequencies, the approximate constants of equations (13) and (15), mf_0^2/sU_0^2 and f_1^2/T, have been evaluated and entered in table 1. The values of mf_0^2/sU_0^2 for vanes A and B are the same to within 10 % for equivalent wire conditions. For vane C (from table 2) the value of the same constant is $(47 \pm 3) \times 10^{-6}$ g/c.c., the conditions on the suspension being equivalent to a high tension in table 1. Considering the accuracy of the measurements involved, better agreement could not have been expected.

The value of f_1^2/T as shown in table 1 evidently tends to a constant value as the tension T is increased. When the tension on the suspension of vane B is low, evidence is obtained of the existence of a second wire frequency $f_2 \sim 2{\cdot}4f_1$. This finding also is consistent with the theory given in § 4.

It was not convenient to test the dependence of f_0 on U in the miniature tunnel because the available range of U was small and wind speed settings were not reproducible. A test of this kind was, however, carried out at a later stage when the 18 in. octagonal tunnel was in use, and the flow in the main stream and in the boundary layer had been calibrated. The vane (vane *C*) was mounted on the vane-head described in § 7 and illustrated in figure 4*b*. On account of the shortness of the suspension wire on the vane head ($l = 1{\cdot}5$ in.) and the tensions used, the wire frequencies were not observable. The vane-head was placed in the free

Description of vane	T (g wt)	f_0 (c/s)	f_1 (c/s)	f_2 (c/s)	mf_0^2/sU_0^2 ($\times 10^6$ g/c.c.)	f_1^2/T (g cm^{-1})	f_2/f_1
Vane *A*							
Silvered coverglass	50	38	176	—	70	620	—
thickness 0·007 in.	100	36	235	—	63	552	—
$2a = 1{\cdot}44$ cm, $s = 0{\cdot}95$ cm	200	35	322	—	59	518	—
$m = 0{\cdot}060$ g, $l = 10$ cm	350	35	418	—	59	499	—
$\rho' = 2{\cdot}11$ mg/cm	500	35	505	—	59	510	—
$U_0 = 37{\cdot}5$ ft./sec	700	35	570	—	59	464	—
Vane *B*							
Silvered coverglass	10	49	122	310	70	1488	2·54
thickness 0·007 in.	20	48	166	387	67	1378	2·33
$2a = 0{\cdot}685$ cm, $s = 0{\cdot}445$ cm	50	48	222	510	67	986	2·30
$m = 0{\cdot}017$ g, $l = 10$ cm	100	46	264	—	62	697	—
$\rho' = 2{\cdot}11$ mg/cm	200	42	382	—	52	730	—
$U_0 = 37{\cdot}5$ ft./sec	500	40	600	—	47	720	—

TABLE 1. Observed frequencies of vanes *A* and *B* under varying tension *T*

	U_0 (ft./sec)	f_0 (c/s)	f_0/U_0
Vane *C*			
Mica foil and metal foil	21	28·5	1·36
thickness 0·0008 in.	24	32	1·33
$2a = 0{\cdot}990$ cm, $s = 0{\cdot}80$ cm	26	33	1·27
$m = 0{\cdot}019$ g, $l = 1{\cdot}5$ in.	28	36	1·28
$\rho' = 43\,\mu$g/cm	33	42·5	1·29
	46	64	1·39
	53	74	1·4
	56	84	1·5
	65	91	1·4
		Mean	$1{\cdot}36 \pm 0{\cdot}08$

TABLE 2. Observed vane frequencies of vane *C* at various wind speeds *U*

stream (velocity U_0) with the vane just outside the boundary layer. A few inches upstream of the vane, a vibrating ribbon was mounted in the boundary layer and used, as by Schubauer & Skramstad (1947), to inject disturbances of known periodicity into the layer. The vane responded to these disturbances and its motion was detected by the electrical method (described in § 6) and recorded photographically from a cathode-ray oscilloscope trace. It was thus possible to determine not only the frequency f_0, but also the frequency response curve of the vane in the neighbourhood of f_0. The dependence of f_0 on U_0 is shown for vane C in table 2. Typical frequency response curves for this vane are shown in figure 3.

The ratio of f_0/U_0 from table 2 is effectively constant and has a mean value of $1{\cdot}36 \pm 0{\cdot}08$ c/ft. (cf. equation (15)).

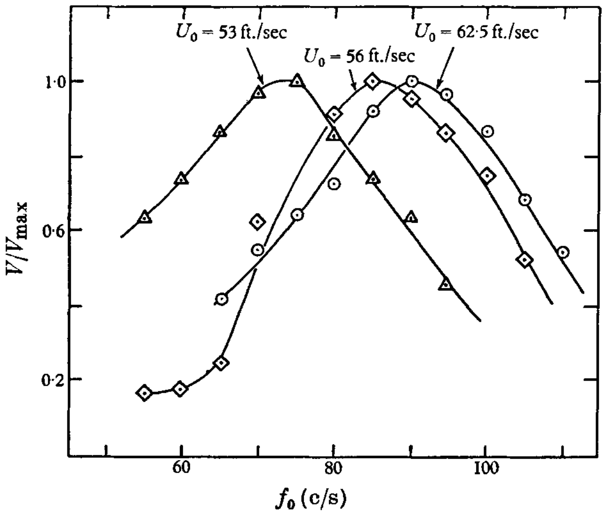

FIGURE 3. Frequency response curves of vane C for three values of U_0.

6. Development of an electrical recording system

It was intended that the vane in its final form would be much lighter than the silvered coverslip used in the preliminary investigations—a thin silvered mica film being at first regarded as the most suitable material. As such films would not be optically flat, the possibility was considered of using the vane as one plate of a capacitor and detecting its movement by measuring changes in capacitance.

The design requirements for such a detecting system were assessed by considering the magnitude of the ratio $\sqrt{(\overline{v^2})}/U_0$ in the boundary-layer fluctuations. At a value of $y/\delta = 0{\cdot}5$, Schubauer & Skramstad (1947) found $\sqrt{(\overline{v^2})}/U_0 = 0{\cdot}3\,\%$. It is probable, however, that this value could be raised to 1 % since Schubauer & Skramstad, working under conditions of low free-stream turbulence, did not require to use maximum disturbances. In accordance with equations (3) and (4) the vane is expected to respond to the first power of v/U, and we assume that its angular deflexion is approximately 20 % of the angular change of stream line, i.e. about $\pm 0{\cdot}002$ radian. Using a square vane with sides of 1·0 cm, mounted at a distance of 0·5 cm from a static plate, the capacitance would be 0·017 $\mu\mu$F with fluctuations of $\pm 0{\cdot}0003\,\mu\mu$F.

An electronic circuit was designed to detect changes of capacitance of this order. The first stage was a 1 Mc/s Clapp (1948) oscillator circuit of moderate Q value, in which the variable vane capacitance produced frequency modulation in accordance with the equation $\delta f = -f(\delta C/2C)$. If in this equation we take $C = 12{\cdot}5\,\mu\mu$F, $\delta C = 3 \times 10^{-4}\,\mu\mu$F, and $f = 10^6$ c/s, then $\delta f = 12$ c/s. Such a frequency change can be detected by amplitude modulation in a high-Q filter circuit if the mean operating frequency of the oscillator coincides with the frequency of maximum slope on the response curve of the filter circuit. A quartz crystal filter circuit with a Q value of 5×10^3 was used, giving a change of voltage amplitude of about 1 % for a frequency change of 1 c/s. To adjust the frequency of the oscillator, a 20 $\mu\mu$F variable capacitor with fine-setting control was employed in parallel with the vane and leads capacitance.

The main advantages of the Clapp circuit are its stability and its low noise level. The circuit has, however, to be used in the presence of the noise and vibration produced by the tunnel, so special precautions must be taken to avoid microphony.

The output of the filter circuit was passed through a detector stage to a low-frequency amplifier and thence to a low-frequency cathode-ray oscilloscope. The oscilloscope trace could be inspected visually, or photographed on 35 mm film by an oscilloscope camera giving film speeds up to 8 ft./sec. A tuning indicator was incorporated in the circuit by connecting a cathode follower to the output line of the filter, and monitoring the cathode follower output with a valve voltmeter. Any drift of the oscillator frequency was indicated by a change in the voltmeter reading.

Absolute calibrations of the electrical recording system were carried out by two methods. The first used stage-by-stage analysis. The second provided a calibration of the complete recording system using a known variable input capacitance in place of the vane. The variable input capacitance was of the form $C_1 + C_2 \sin \omega t$, where ω was variable, and the values of C_1 and C_2 were comparable, respectively, to the mean capacitance of the vane, and the amplitude of its capacitance change.

In the overall calibration, the sensitivity was found to be constant for frequencies of capacitance change from 50 to 200 c/s. The voltage output of the recording system is proportional to V_{max}, the maximum voltage output of the filter circuit (as recorded on the monitoring valve voltmeter); the normal value of this maximum voltage in our circuit is 20 V. The stage-by-stage calibration gave the sensitivity of the system as $4{\cdot}5 \pm 0{\cdot}5$ V per $10^{-3}\,\mu\mu$F and the overall calibration gave $3{\cdot}4 \pm 0{\cdot}7$ V per $10^{-3}\,\mu\mu$F. The agreement between the two determinations of sensitivity was considered to be satisfactory, and the 'best' value of the sensitivity was taken to be 4 (or $0{\cdot}2V_{max}$) V per $10^{-3}\,\mu\mu$F.

7. Design of the support system and the vane-head

It was originally thought that the flat plate itself might be used as the second plate of the vane capacitor. Vibrations at fan-blade frequency were, however, found to occur on the plate with an amplitude of 10^{-4} in. As these frequencies are comparable with those of the Tollmien–Schlichting waves, it was decided to eliminate tunnel vibration as completely as possible from the signal given by the vane capacitor.

Accordingly, a new support system for the vane was developed. A rigid 2 in. angle-iron frame was built round the tunnel and was supported (independently of the tunnel supports) on concrete blocks insulated with felt pads from the floor. Two rods mounted about 3 ft. 6 in. apart on this frame passed into the working section through slightly oversize holes in the tunnel wall. The rigidity of the carriage rails mounted inside the tunnel on these rods had to be limited in order to reduce the area of blockage in the working section. By using a $\frac{3}{8}$ in. square section rod as a carriage rail and tapering its upstream end, the area of blockage was reduced to 0·12 sq.in. in the 134 sq.in. area of the half section of the tunnel. The carriage was a box spar running on the square rod, two faired strips supporting the boom. The carriage support was found to be sufficiently heavy to give no vibration trouble. The micrometers controlling the y-traverse were mounted outside the tunnel. A sketch of the system is shown in figure 4a.

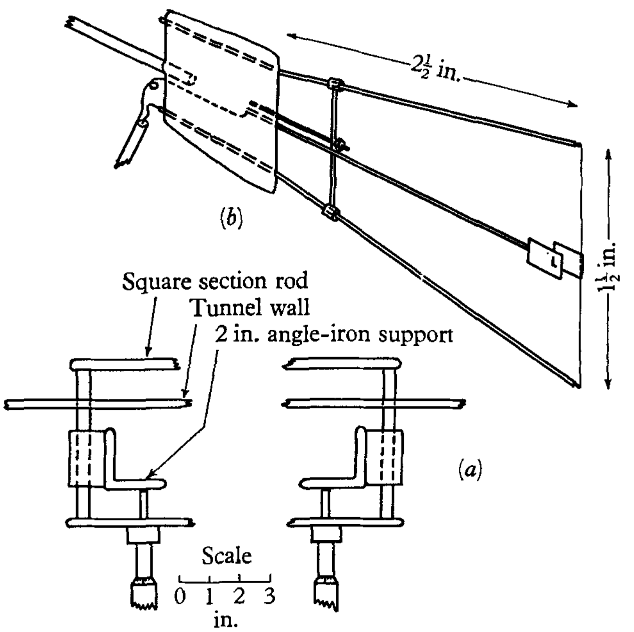

FIGURE 4. (a) Support system of the vane-head, mounted independently of the tunnel supports; y-traverse micrometers are shown. (b) Sketch of the vane-head.

A 'vane-head' was now developed, incorporating the vane and a static plate (see figure 4b). The body of the head was made from $\frac{1}{4}$ in. ebonite sheet, streamlined and polished. Two hypodermic tubes lined with fine glass tubing were mounted in the body and projected $2\frac{1}{2}$ in. forward from it; they were $\frac{3}{4}$ in. apart where they entered the body and $1\frac{1}{2}$ in. apart at their forward ends. A suspension of 0·001 in. diameter Nichrome wire was threaded through the glass tubing and anchored at the rear of the body, one end being soldered to an electrical contact. A strut of fixed length sliding under fine screw control on the hypodermic tubes determined the tension in the suspension. The static plate was of 0·003 in. thickness and was supported on a fine boom having a circular section of 0·05 in. diameter at the body, tapering to a thin rectangular section at its junction with the plate. The separation between the static plate and the vane was normally 0·15 in.—sufficient to ensure that the static plate would lie outside the boundary layer. The boom and the suspension wire were connected to the two leads of the coaxial cable. Some difficulty was experienced with the coaxial cable—partly at different distances from the leading edge. If a disturbance at the vane frequency was set up naturally in the boundary layer, it would decrease in amplitude till it reached branch I of the neutral curve, increase thereafter till it reached branch II and then decrease once more. It might therefore be possible to find the x-position of maximum disturbance of the vane, and so to determine a point on or near branch II of the neutral curve.

From the neutral curve data given by Schlichting (1933) it was deduced that, using air speeds of 50–70 ft./sec, and x-positions of 2–3 ft. from the leading edge, vanes with frequencies of 200–300 c/s should be suitable. Vanes with these frequencies were constructed and moved down the flat plate in the boundary layer. They responded to the random disturbances due to turbulence and their motion was therefore irregular. Quantitative work was not possible, but qualitatively there was evidence of a limited region of increased response. It was, however, preferable at this stage to work with a more constant signal.

Artificial oscillations were therefore injected into the boundary layer by the 'ribbon' technique of Schubauer & Skramstad (1947). A phosphor-bronze ribbon of 0·1 in. × 0·001 in. cross-section was suspended under tension in the boundary layer, the vibrating length of the ribbon being determined by bridges, 0·006 in. high, resting against the flat plate. A localized magnetic field was produced at the centre of the ribbon, using a small 'Eclipse' permanent magnet mounted on the back of the flat plate. A variable frequency $\frac{1}{2}$ W signal-generator supplied current to the ribbon. To obtain a sufficient response, it was found necessary to use the ribbon nearly on resonance with the signal-generator frequency. Special weights of 50, 100, and 150 g wt. were cast in stream-line shape for suspending on the ribbon, and the tension and free length of the ribbon were adjusted so that the ribbon frequency was somewhat higher than the frequency—determined by the vane frequency—at which the signal-generator was to operate.

An exact replica of the ribbon arrangement was set up outside the tunnel on $\frac{1}{4}$ in. Perspex plate so that, using a microscope, the amplitude of the ribbon vibrations could be measured as a function of the voltage output of the signal-generator. The equality of the resonance frequencies of the two ribbons could be tested by ear. An investigation was now made of the effect of the amplitude, frequency, and position of the vibrating ribbon on the position of the transition front in the boundary layer. When the ribbon was placed 1 ft. 2 in. from the leading edge, ribbon amplitudes greater than 0·006 in. brought the transition front forward, but when the ribbon was at 1 ft. 6 in. from the leading edge, it had no effect on the transition front. The latter position was therefore used in all further work.

As the motion of the ribbon could now be described both in frequency and amplitude, and the velocity v imparted to the air stream could therefore be determined to a good approximation, an attempt was made to measure the minimum signal to which the vane would give a recognizable response. A vane with a natural frequency of 120 c/s was placed in the boundary layer 6 in. downstream of the ribbon and at $y/\delta = 0{\cdot}5$. The air speed was adjusted to 70 ft./sec at which speed the net amplification of the disturbance was expected to be zero. An easily indentifiable signal was then obtained for a value of $\sqrt{(\overline{v^2})}/U_0 = 0{\cdot}005$.

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This observation is consistent with the known free-stream turbulence level of the tunnel, $\sqrt{(\overline{v^2})}/U_0 = 0{\cdot}003$.

Using the ribbon and vane, the progress of the laminar oscillations in the boundary layer were traced downstream at constant y/δ. The best procedure was found to be to vary the input voltage of the signal-generator so as to maintain a constant 'well-defined' signal from the vane. The 'well-defined' signal was chosen arbitrarily, but was sufficient to mask the background signal due to turbulence. An alternative procedure was used when photographic recordings were made; a constant output was maintained from the signal-generator and the variable vane response was measured from the record. Both methods gave essentially the same results.

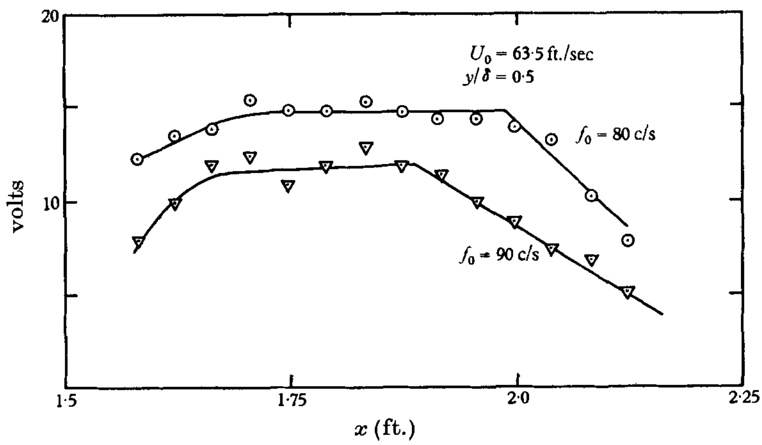

FIGURE 5. Signal-generator voltage required to produce constant amplitude of vane response versus distance of the vane from the leading edge of the flat plate. Traverse through branch I of the Tollmien–Schlichting neutral curve.

Figure 5 shows typical results from two different vanes, obtained by the first of these methods when the vane was traversed through branch I of the Tollmien–Schlichting neutral curve. The voltage output for a fixed vane amplitude is plotted as ordinate against the x-position of the vane. Figure 6 similarly shows typical results obtained by traversing a vane through branch II of the neutral curve.

In each of these graphs the plotted points lie fairly definitely on two straight lines. The transition from one slope to the other is probably smooth, but to make an estimate of the position of the neutral curve, the two straight lines have been extrapolated to meet at a point. In figure 5, there appears to be an extended region of negligible amplification or damping followed by one of amplification. The transition from one region to the other is taken as indicating a crossing of branch I of the neutral curve. In figure 6, a region of large amplification is followed by one of much smaller amplification, and the transition is interpreted as a crossing of branch II of the neutral curve.

From a series of such experiments, a number of neutral points has been obtained. These are shown in figure 7, where they are compared with the theoretical curves given by Schlichting (1933) and by Shen (1954). The presence in our tunnel of a fairly high level of turbulence means that at branch I of the neutral curve, amplification of natural oscillations may contribute to the gentle slope in the almost flat portion of the curve in figure 5. Similarly, natural oscillations may contribute to the residual slope in the later portion of the graph in figure 6. The

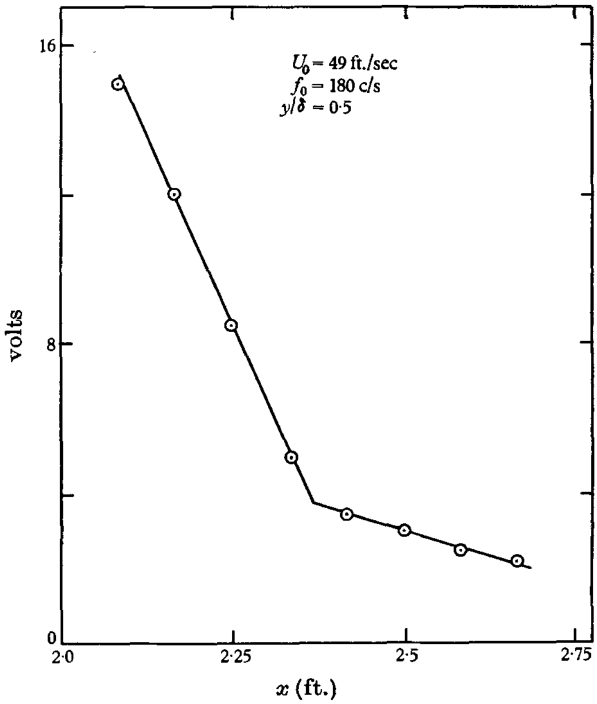

FIGURE 6. Signal-generator voltage required to produce constant amplitude of vane response versus distance of the vane from the leading edge of the flat plate. Traverse through branch II of the Tollmien–Schlichting neutral curve.

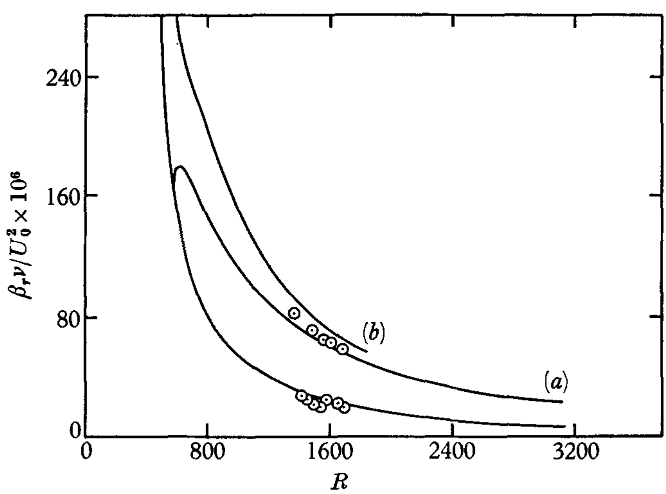

FIGURE 7. Theoretical neutral curves (a) due to Schlichting, (b) due to Shen, and experimental points on the curve derived from graphs similar to figures 5 and 6.

fair agreement of our observations in figure 7 with the theoretical curve indicates, however, that the method we have used has been successful in detecting the position of the neutral curve notwithstanding the presence of the relatively high free-stream turbulence.

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